Enrollment No: ____________________ Exam Seat No: ____________________

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name : Differential and Integral Calculus

Subject Code : 4SC04MTC1 **Branch: B.Sc.(Mathematics,Physics)**

Semester : 4 **Date : 10/05/2016** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Form the partial differential equation from $z = ax + by$. **(02)**

b) Define: node. **(02)**

c) State Green's theorem. **(02)**

d) If $F = xi + yj + zk$, then show that F is irrotational. **(02)**

e) State Stoke's theorem. **(02)**

f) If $F = yz + zx + xy$ then find ∇F. **(02)**

g) Show that $F = 3y^4 i + 4x^3 z^2 j - 3x^2 y^2 k$ is solenoidal. **(02)**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Evaluate $\int_c \bar{F}.dR$, where $\bar{F} = (x^2 + y^2)i - 2xyj$ and C is the rectangle in the xy −plane bounded by $y = 0, x = a, y = b, x = 0$. **(07)**

b) State the necessary and sufficient conditions for any point (x, y) on $f(x, y) = 0$ to be a double point. Find the position and nature of the double points on the curve $a^4 y^2 = x^4(2x^2 - 3a^2)$. **(07)**

Q-3 **Attempt all questions** **(14)**

a) Show that the curve $(a^2 + x^2)y = a^2 x$ has three points of inflexion. **(07)**

b) Prove that for vector function F, $\nabla \times (\nabla \times F) = \nabla(\nabla.F) - \nabla^2 F$. **(07)**

Q-4 **Attempt all questions** **(14)**

a) Change the order of integration $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2}{\sqrt{(y^4 - a^2x^2)}}\, dydx$ and hence evaluate the same. **(07)**

b) Verify Stoke's theorem for the vector field $F = (2x - y)i - yz^2 j - y^2 zk$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy −plane. **(07)**

Q-5		**Attempt all questions**	**(14)**
	a)	(i) Form the partial differential equation from $F(x+y+z, x^2+y^2+z^2)=0$. (ii) Find the differential equation of the set of all spheres whose centers lie on the z −axis	**(07)**
	b)	Find the directional derivative of $\phi = x^2 - y^2 + 2z^2$ at the point $P(1,2,3)$ in the direction of the line PQ where Q is the point $(5,0,4)$. In what direction will it be maximum? Find the maximum value of it.	**(07)**
Q-6		**Attempt all questions**	**(14)**
	a)	Define: gradient. Prove that $\nabla r^n = nr^{n-2}\bar{r}$, where $\bar{r} = xi + yj + zk,\ r = \lvert\bar{r}\rvert$.	
	b)	Evaluate $\iint_R (x+y)^2\, dxdy$, where R is the parallelogram in the xy −plane with vertices $(1,0), (3,1), (2,2), (0,1)$ using the transformation $u = x + y$ and $v = x - 2y$.	**(07)**
Q-7		**Attempt all questions**	**(14)**
	a)	(i) Find the equations of the tangent plane and the normal to the surface $z^2 = 4(1 + x^2 + y^2)$ at $(2,2,6)$. (ii) Solve: $yz\,p + zx\,q = xy$.	**(07)**
	b)	Verify Green's theorem for $\int_C [(xy + y^2)dx + x^2dy]$, where C is bounded by $y = x$ and $y = x^2$.	**(07)**
Q-8		**Attempt all questions**	**(14)**
	a)	Prove that $(0,-2), (2,0)$ and $(-2,0)$ are node of the curve $x^4 - 4y^3 - 12y^2 - 8x^2 + 16 = 0$.	**(07)**
	b)	(i) Evaluate $\int_0^\infty \int_0^\infty \mathrm{e}^{-(\mathrm{x}^2+\mathrm{y}^2)}$ dxdy, by changing to polar coordinates. (ii) Evaluate $\int_{-1}^{1}\int_0^z \int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$.	**(07)**